John Wollaston Anglican Community School

MATHEMATICS
METHODS
UNIT 1
Section Two:
Calculator-assumed

If required by your examination administrator, please place your student identification label in this box

Student number: In figures

In words _____

Your name _____

Time allowed for this section
Reading time before commencing work: ten minutes
Working time: one hundred minutes

Materials required/recommended for this section
To be provided by the supervisor
This Question/Answer booklet
Formula sheet (retained from Section One)

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in this examination

Important note to candidates
No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	52	35
Section Two: Calculator-assumed	13	13	100	98	65
				Total	100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet preferably using a blue/black pen.
   Do not use erasable or gel pens.

3. You must be careful to confine your answer to the specific question asked and to follow any instructions that are specified to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

Markers use only		
Question	Maximum	Mark
9	6	
10	8	
11	8	
12	8	
13	8	
14	8	
15	7	
16	8	
17	7	
18	6	
19	8	
20	8	
21	8	
S2 Total	98	
S2 Wt (×0.6633)	65%	

SN044-132-3

DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF

Section Two: Calculator-assumed **65% (98 Marks)**

This section has **thirteen (13)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

Question 9 **(6 marks)**

(a) The points A and B have coordinates $(4, -6)$ and $(5, 8)$ respectively. If B is the midpoint of A and C, determine the coordinates of C. (3 marks)

(b) The points D and E have coordinates $(5p, -q)$ and $(2q, 3p)$ respectively, where p and q are constants. Determine the value of p and the value of q if the midpoint of D and E is at $(21, 17)$. (3 marks)

Question 10 **(8 marks)**

A positive integer less than 10 is chosen at random.

The outcome sets for events B, C and D are $B = \{1, 4, 9\}$, $C = \{1, 6, 9\}$ and $D = \{5, 6, 7\}$.

(a) List the following sets:

(i) $C \cap D$. (1 mark)

(ii) $B \cup C \cup D$. (2 marks)

(iii) $(B' \cap C')'$. (1 mark)

(b) Determine

(i) $\text{n}(C \cap D')$. (1 mark)

(ii) $P(B \cap D)$. (1 mark)

(iii) $P\big(B'\big|(C \cup D)\big)$. (2 marks)

Question 11 **(8 marks)**

In an experiment, the sound intensity, S, can be modelled by $S(x) = 3.6 - 2.4x + 1.5x^2 - 0.2x^3$, where x is the distance from the sound source in metres and $0 \leq x \leq 6$.

(a)   Determine S when $x = 3$.   (1 mark)

(b)   Draw the graph of $y = S(x)$ on the axes below.   (4 marks)
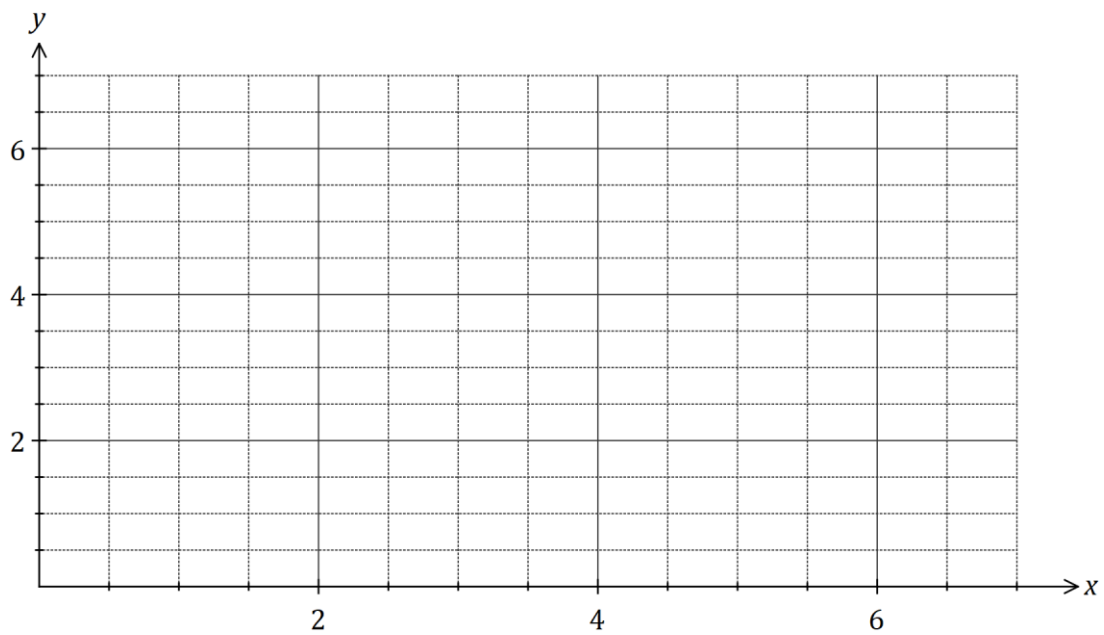
(c)   Determine the equation of the straight line L that passes through the x-intercept and the y-intercept of the graph of $y = S(x)$.   (2 marks)

(d)   Determine the coordinates of the point of intersection of L with the graph of $y = S(x)$ where $x > 0$ and $y > 0$.   (1 mark)

Question 12 **(8 marks)**

A random sample of 121 passengers arriving at an airport were asked to complete a brief survey. They were asked to categorise their main place of residence as Australia or overseas and the main purpose of their travel as work, holiday or other. It was found that

- half of the 84 passengers who resided overseas were on holiday
- 14 passengers were on holiday and resided in Australia
- of the 27 who were travelling for other reasons, 11 more resided overseas than in Australia.

(a) Use the above information to complete the two-way table below. (3 marks)

	Work	Holiday	Other	Total
Australia				
Overseas				84
Total			27	121

(b) If one passenger was selected at random from those surveyed, determine the probability that

(i) the main purpose of their travel was work. (1 mark)

(ii) they resided overseas, given that the main purpose of their travel was work. (1 mark)
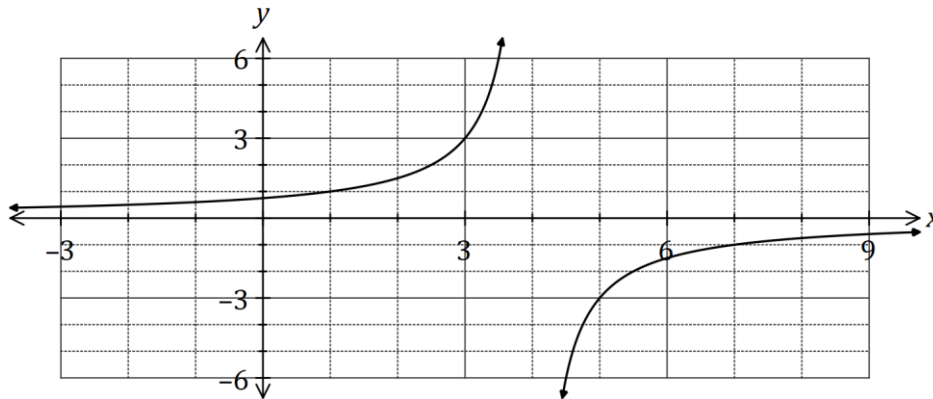
(iii) the main purpose of their travel was work, given that they resided in Australia. (1 mark)

(c) Explain whether the survey indicates that purpose of travel appears to be independent of main place of residence for these passengers. (2 marks)

Question 13 **(8 marks)**

The graph of $y = f(x)$ is shown below where $f(x) = \dfrac{c}{a-x}$.

(a) State the value of the constant a and the value of the constant c. (3 marks)

(b) The hyperbola shown above has two asymptotes. State their equations. (2 marks)

(c) Describe how to transform the graph of $y = f(x)$ to obtain the graph of $y = f(x-3)$ and state the domain and range of the transformed function. (3 marks)

Question 14 **(8 marks)**

Two events, A and B, have probabilities $P(A) = 0.4$ and $P(B) = 0.65$.

(a) Determine $P(A \cap B)$ in each of the following cases:

(i) A and B are independent. (1 mark)

(ii) $P(A \cup B) = 0.8$. (2 marks)

(iii) $P(A|(A \cup B)) = \dfrac{4}{9}$. (3 marks)

(b) Is it possible that A and B are mutually exclusive events? Explain your answer. (2 marks)

Question 15 **(7 marks)**

An **obtuse** angled triangle ABC has $a = 36$ cm, $c = 52$ cm and an area of 748 cm^2.

(a) Sketch a triangle to show this information. (1 mark)

(b) Determine the size of $\angle B$. (2 marks)

(c) Show that $b \approx 79$ cm. (2 marks)
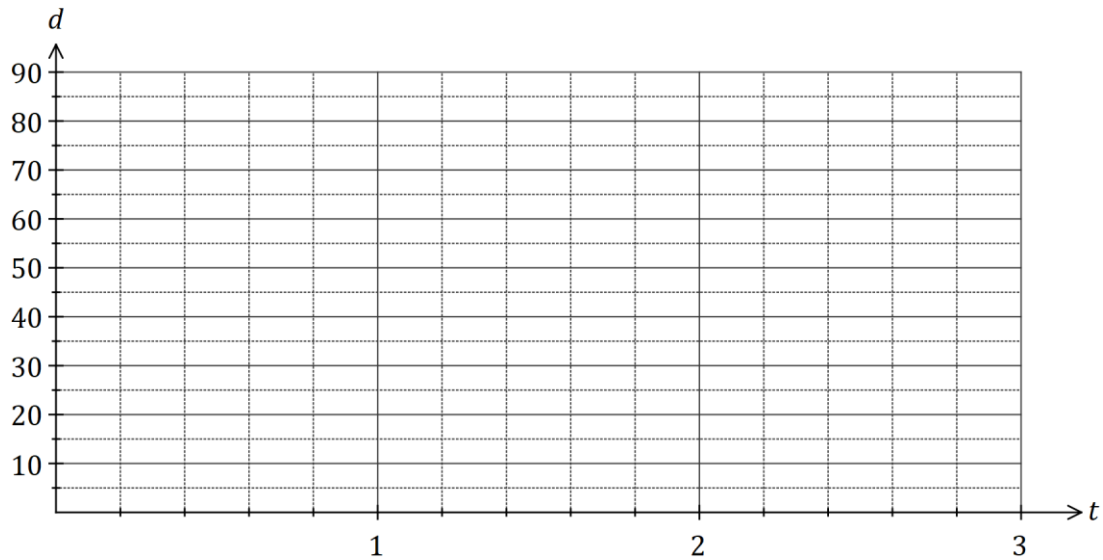
(d) Show that $\angle C \approx 32°$. (2 marks)

Question 16   **(8 marks)**

A small weight, attached to the bottom of a spring, oscillated up and down. The distance, d cm, of the weight from the top of the spring after t seconds can be modelled by

$$d = 45 + 35 \sin\left(\frac{3\pi t}{4}\right)$$

(a)   Sketch the graph on the axes below for $0 \leq t \leq 3$.   (4 marks)

(b)   Mark on your graph point M, where the weight is 40 cm from the top of the spring and moving downwards.   (1 mark)

(c)   Determine

         (i)   the maximum distance of the weight from the top of the spring.   (1 mark)

         (ii)   the time taken for the weight to first return to its initial position.   (1 mark)

         (iii)   the distance moved by the weight between $t = 1$ and $t = 2$.   (1 mark)

Question 17 **(7 marks)**

An examination consisted of two papers, one of which was much harder than the other. 15% of candidates gained a distinction in the first paper (event A) and 5% gained a distinction in the second paper (event B) whilst 82% did not gain a distinction in either paper.

(a)   Determine the probability that a randomly chosen candidate

    (i)   gained a distinction in both papers. (2 marks)

    (ii)   gained a distinction in one paper but not the other. (2 marks)

    (iii)   gained a distinction in the first paper given that they gained a distinction in the second. (1 mark)

(b)   State, with justification, whether events A and B are independent. (2 marks)

Question 18 **(6 marks)**

Let $a = \sin 50°$ and $b = \cos 100°$.

Give your answers to the following in terms of a and/or b.

(a) Write down an expression for

(i) $\sin 130°$. (1 mark)

(ii) $\cos 80°$. (1 mark)

(b) Determine an expression for $\cos 130°$. (3 marks)

(c) Determine an expression for $\tan 130°$. (1 mark)

Question 19 **(8 marks)**

(a) The equation of the axis of symmetry for the graph of $y = 3x^2 + 6x + 7$ is $x = k$.
Determine the value of k, using a method that does not refer to the graph of the parabola.
(2 marks)

(b) A parabola with equation $y = ax^2 + bx + c$ has a turning point at $(6, -5)$ and passes
through the point $(-2, -37)$. Determine the value of a, the value of b and the value of c.
(3 marks)

(c) Determine the value of the discriminant for the quadratic equation $16x^2 - 24x + 9 = 0$ and
use it to explain how many solutions the equation $(x + 1)(16x^2 - 24x + 9) = 0$ will have.
(3 marks)

Question 20 **(8 marks)**

A shelf held a collection of 18 different books, of which 7 were cookbooks, 3 were dictionaries and the rest were travel guides.

A random selection of 5 books is to be made from the shelf.

(a) Determine the number of ways

(i) this can be done. (2 marks)

(ii) a selection can be made that will not contain any dictionaries. (2 marks)
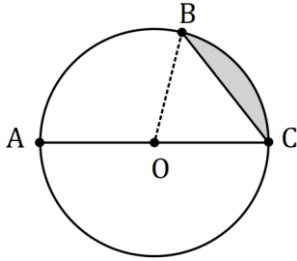
(b) Determine the probability that

(i) the selection will only contain cookbooks. (2 marks)

(ii) the selection will contain exactly one cookbook given that it does not contain any dictionaries. (2 marks)

Question 21   **(8 marks)**

(a)   The circle shown has centre O and diameter AC of length 50 cm. Determine the shaded
        area given that $2 \times \angle AOB = 3 \times \angle BOC$.   (4 marks)

(b)   A sector of a circle has a perimeter of 112 cm and an area of 735 cm². Determine the
        radius of the circle.   (4 marks)

End of questions

Supplementary page

Question number: _____

Supplementary page

Question number: _____

Supplementary page

Question number: _____

Supplementary page

Question number: _____